How many participants do I need

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Digital Thermometers Readings

Sequential Analysis

- Traditional Sampling: sample size is a pre-defined fixed number:
  - Pre-specify the sample size, e.g. 30;
  - Take the sample;
  - Statistical analysis.

- Sequential Sampling: sample size is a random number:
  - Take the sample and conduct analysis sequentially;
  - Stop if some "stopping rule" is satisfied.

History of Sequential Analysis

The method of sequential analysis is first attributed to Abraham Wald while at Columbia University's Statistical Research Group as a tool for more efficient industrial quality control during World War II.

Key step: **Stopping Rule!**

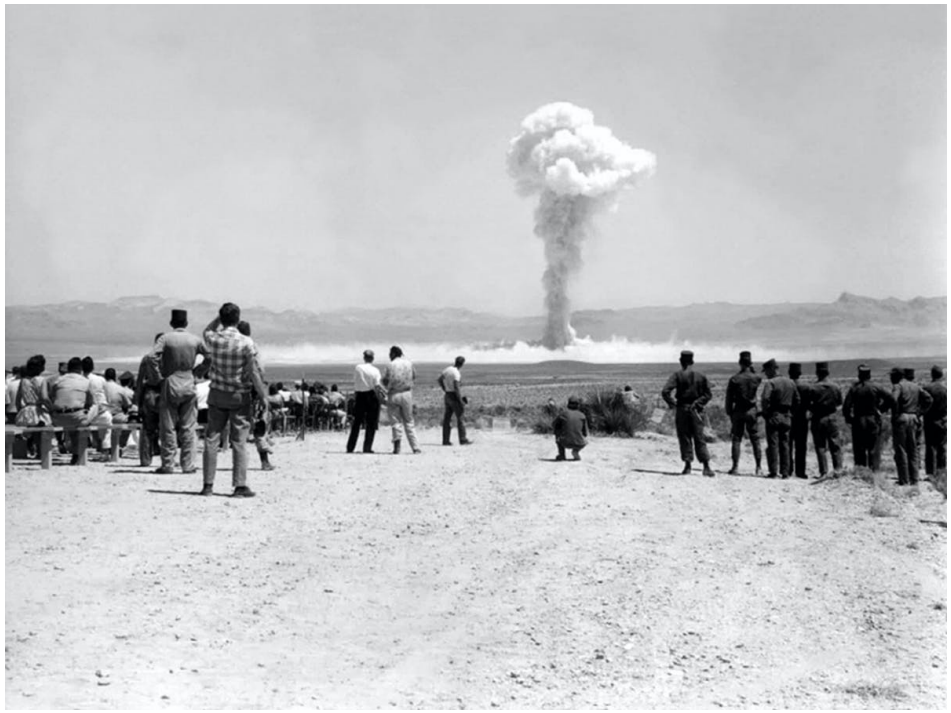


Figure: Nuclear Test in World War II.

Application 1: Control Charts

Control charts, also known as Shewhart charts (after Walter A. Shewhart) or process-behavior charts, or process-control charts, was invented by Walter A. Shewhart working for Bell Labs in the 1920s. It is one of the seven basic tools of quality control.

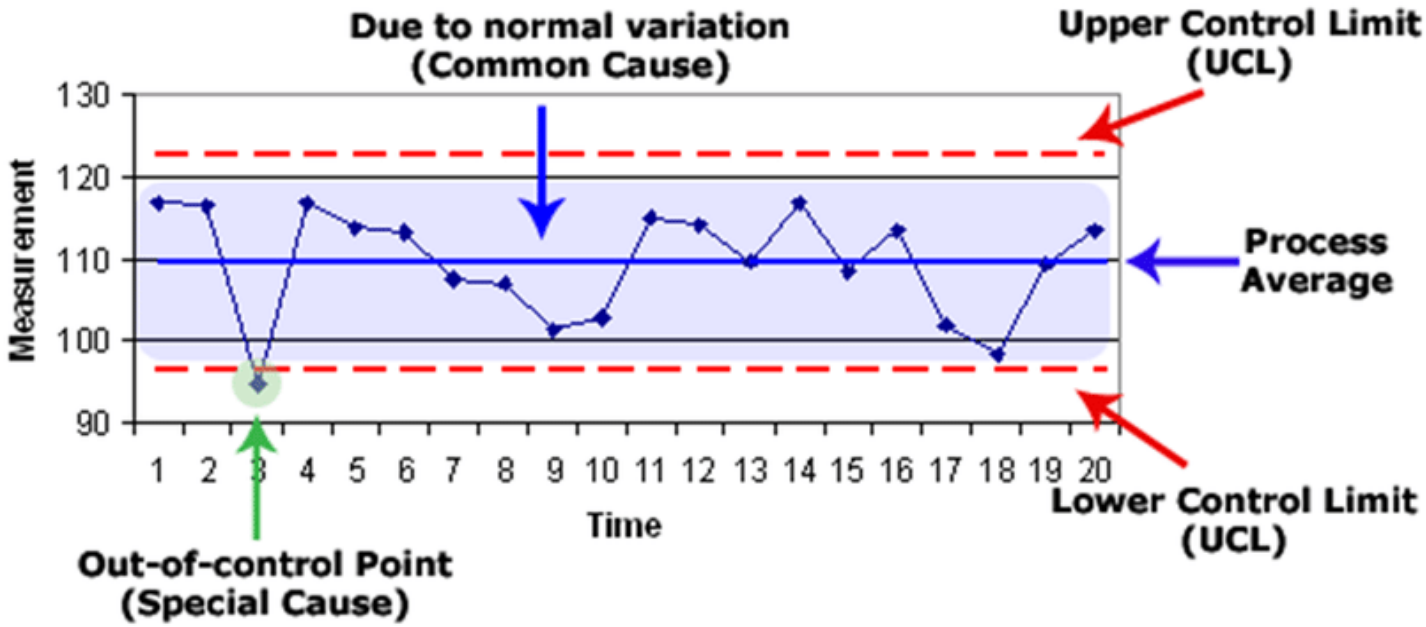


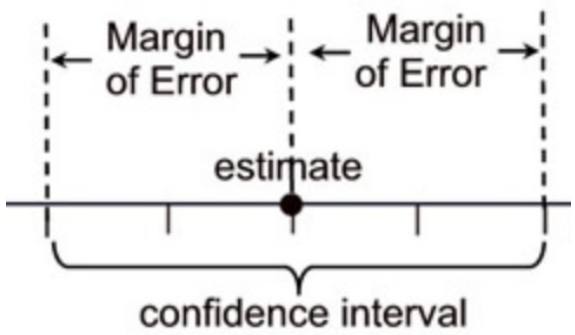
Figure: Process Control Chart

Normal range: Average $\pm$ Margin of Error

Application 2: Fixed Size Interval Estimation

I want to estimate the battery life of iPhone 12 mini.

1. Charge the battery to 100% every morning and then letting it run all the way down until the iPhone shuts off.
2. Record the battery lifespan.
3. Repeat it for several days.
4. How many days do I need to get a precise estimated range of battery life? Say ?hrs $\pm$ 30mins.

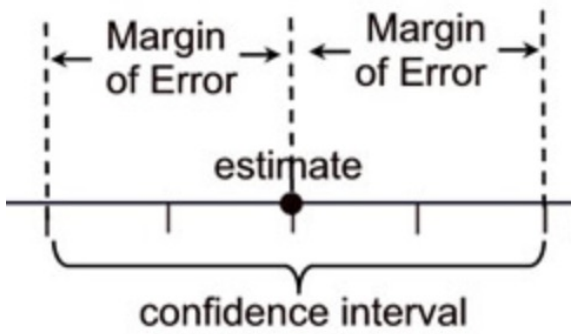


Application 2: Fixed Size Interval Estimation

The confidence interval has the following format:

$$[\text{Sample Mean} \pm \text{Margin of Error}]$$

$$\left[\bar{X} - 1.96\frac{s}{\sqrt{n}}, \bar{X} + 1.96\frac{s}{\sqrt{n}}\right]$$



s: sample standard deviation.
As n increases, what will happen to the margin of error?

Simulation

Size	Data	Interval
4	10.9, 12.6, 11.3, 13.7	12.13 ± 2.03
5	10.9, 12.6, 11.3, 13.7, 10.8	11.86 ± 1.56
6	10.9, 12.6, 11.3, 13.7, 10.8, 11.6	11.82 ± 1.18
7	10.9, 12.6, 11.3, 13.7, 10.8, 11.6, 13	11.99 ± 1.04
...	...	...
23	10.9, 12.6, 11.3, ... , 12.0, 10.0	11.92 ± 0.58
24	10.9, 12.6, 11.3, ... , 12.0, 10.0, 11.0	11.90 ± 0.49
25	10.9, 12.6, 11.3, ... , 12.0, 10.0, 11.0, 11.5	11.76 ± 0.47

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23	10.9, 12.6, 11.3,..., 12.0, 10.0	11.92 ± 0.58
24	10.9, 12.6, 11.3,..., 12.0, 10.0, 11.0	11.90 ± 0.49
25	10.9, 12.6, 11.3, ... , 12.0, 10.0, 11.0, 11.5	11.76 ± 0.47

- Start with a small pilot sample, say, a sample of size 4;
- Calculate the margin of error;
- If the margin of error > 0.5, sample one more data, recalculate the margin of error;
- Stop the sampling the first time "margin of error<0.5" is satisfied.

Application 2: Fixed Size Interval Estimation

I want to estimate the number of deer on campus using capture - recapture method:

- M: initial sample size captured and marked;
- n: second sample size recaptured independently;
- m: number of sample in the recaptured one that marked;
- N: total population size

Since the proportion of the marked subjects in the recaptured sample is likely to be about the same as the first sample in the whole population:

$$\frac{m}{n} \approx \frac{M}{N}$$

Application 2: Fixed Size Interval Estimation

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- m: number of sample in the recaptured one that marked;
- N: total population size

$$\hat{N} = \frac{n}{m} M$$

with the confidence interval

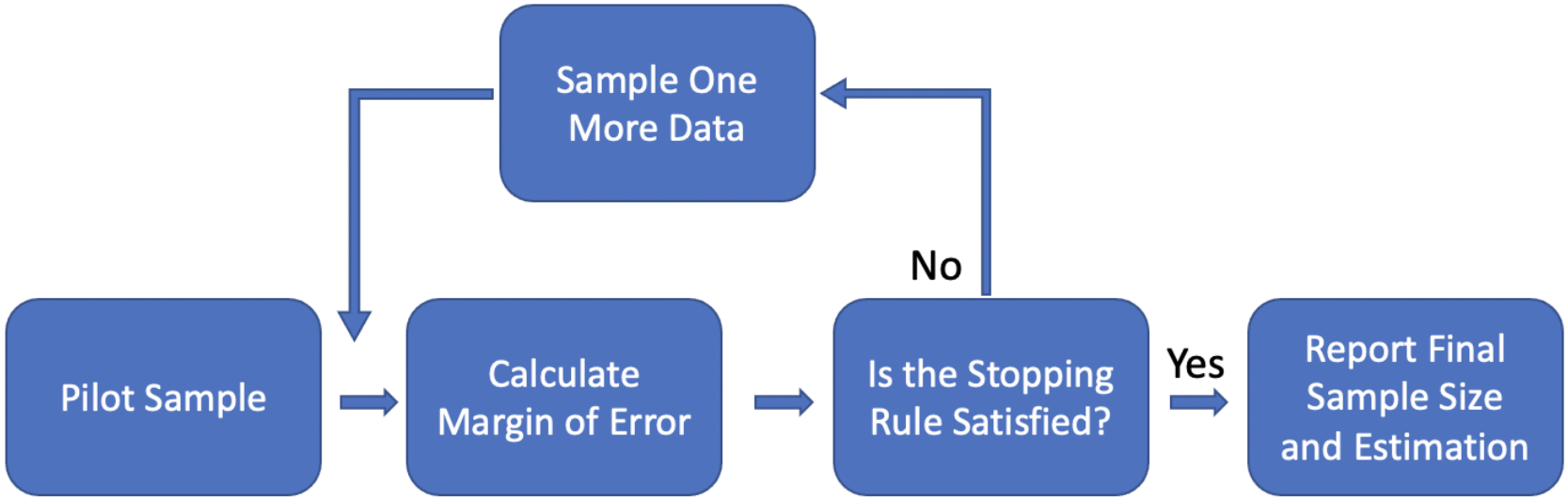
$$\hat{N} \pm \text{ Margin of Error}$$

Simulation

Suppose we capture and mark $M = 10$ deer at Denison.

n	m	$\hat{N}$	Confidence Interval
10	2	50	50 ± 55.43
11	2	55	55 ± 61.62
12	2	60	60 ± 67.89
13	3	43	43 ± 35.98
...			
35	6	58	58 ± 26.87
...			
58	9	64	64 ± 12.23

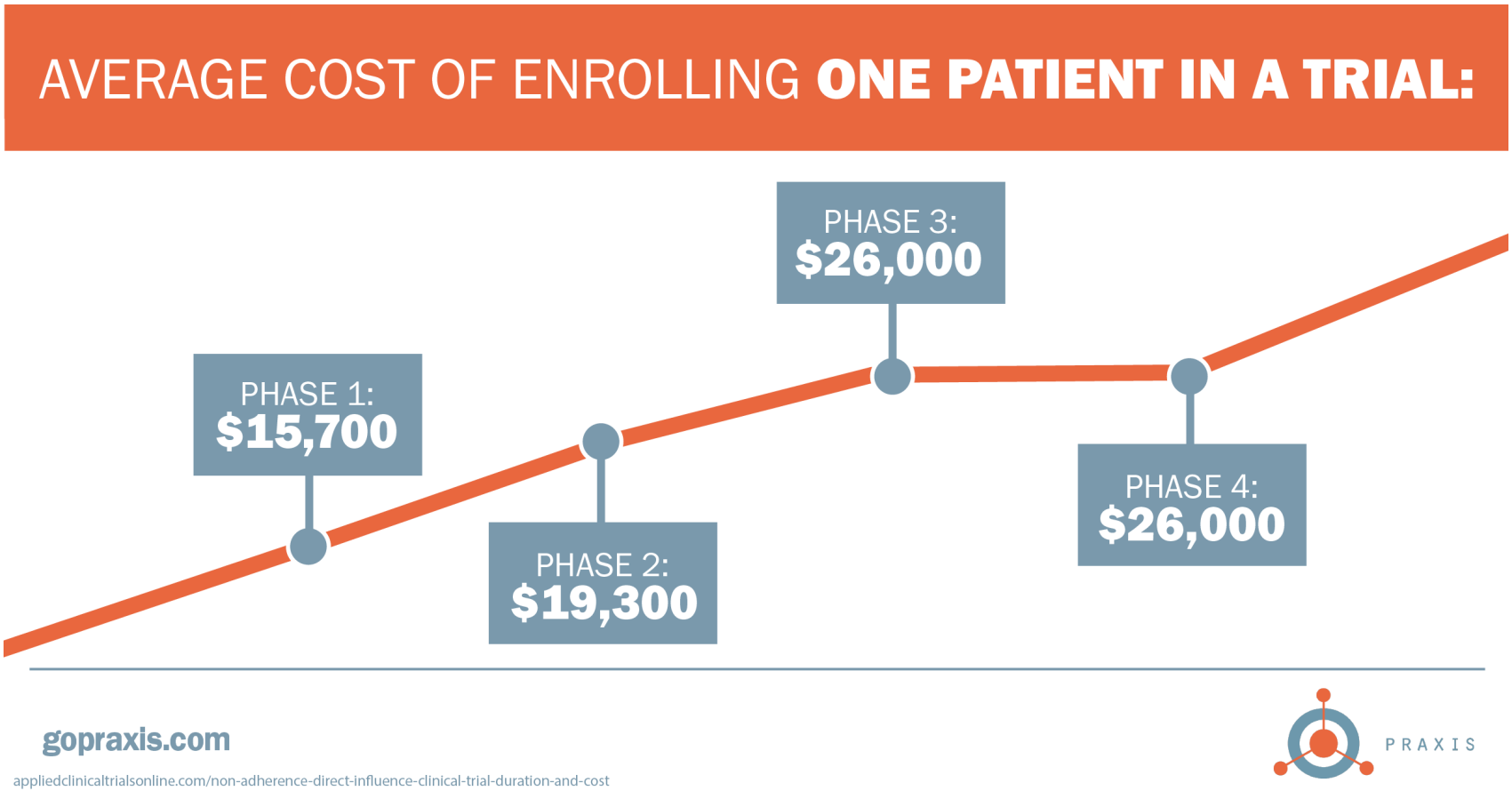
Flow Chart



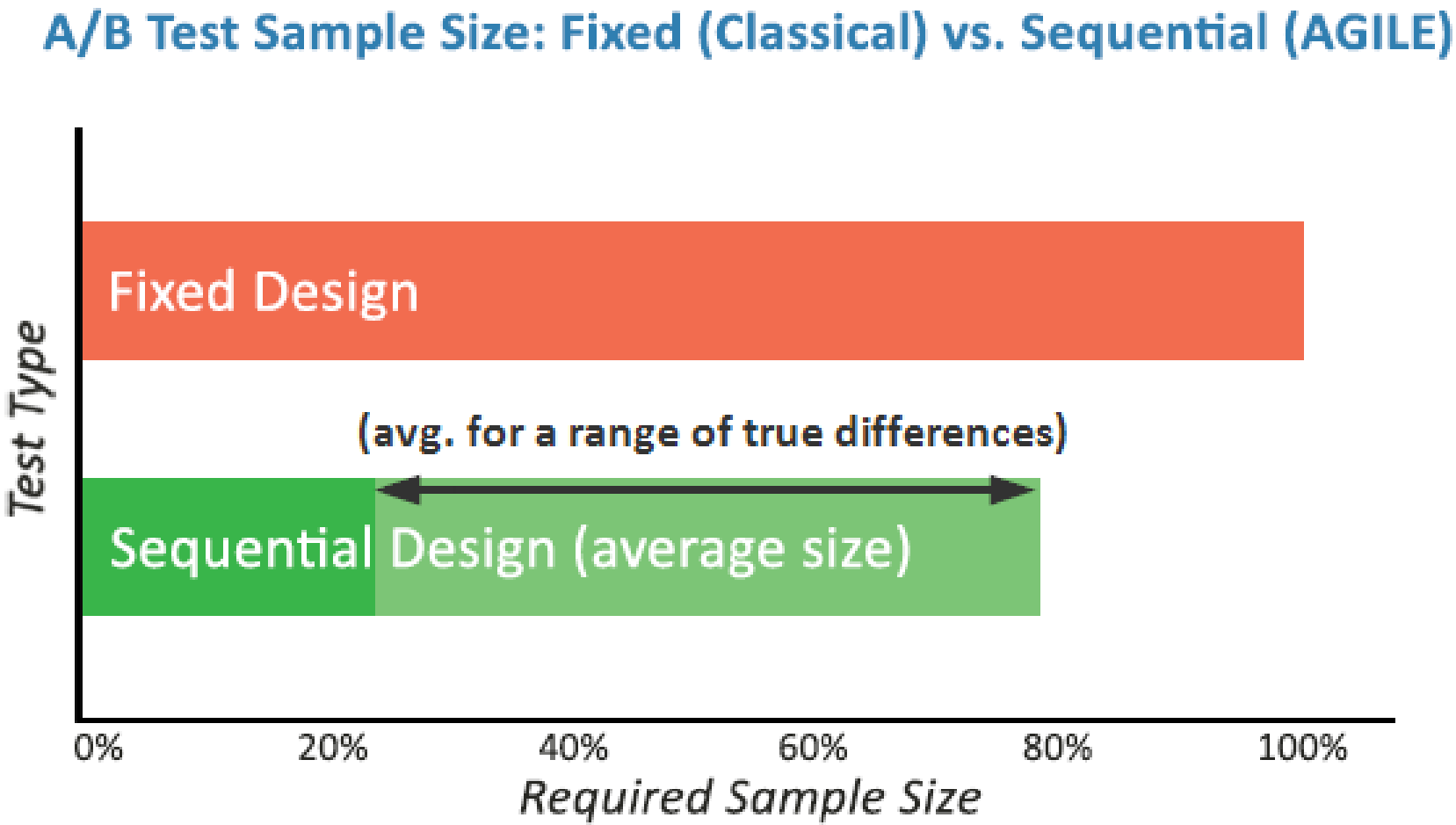
Possible stopping rule:

- Go beyond tolerance zone;
- Margin of error $< e$;
- Statistical power $> q$;
- ...

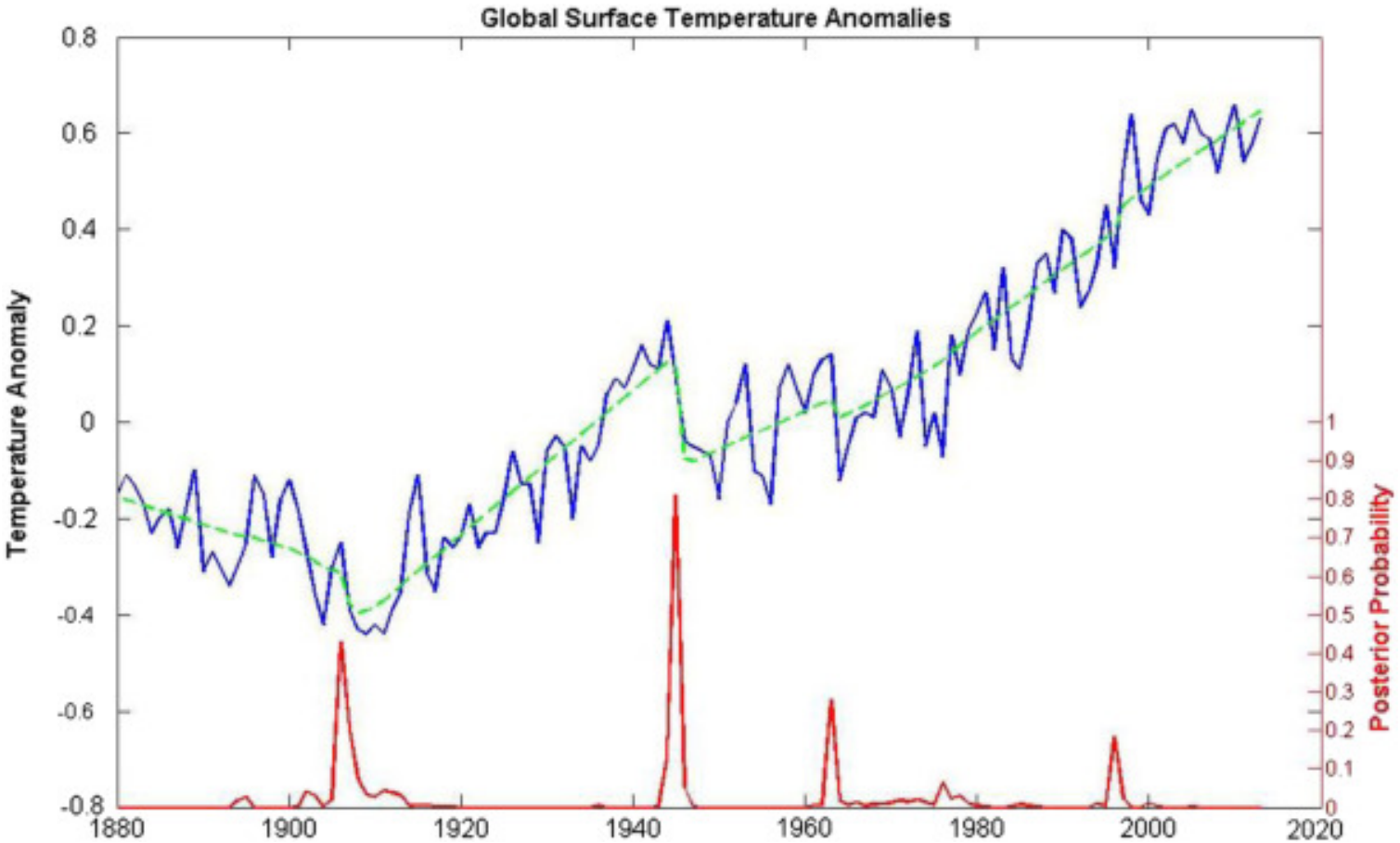
Industrial Application: Sequential Adaptive Design



Industrial Application: Sequential A/B Test

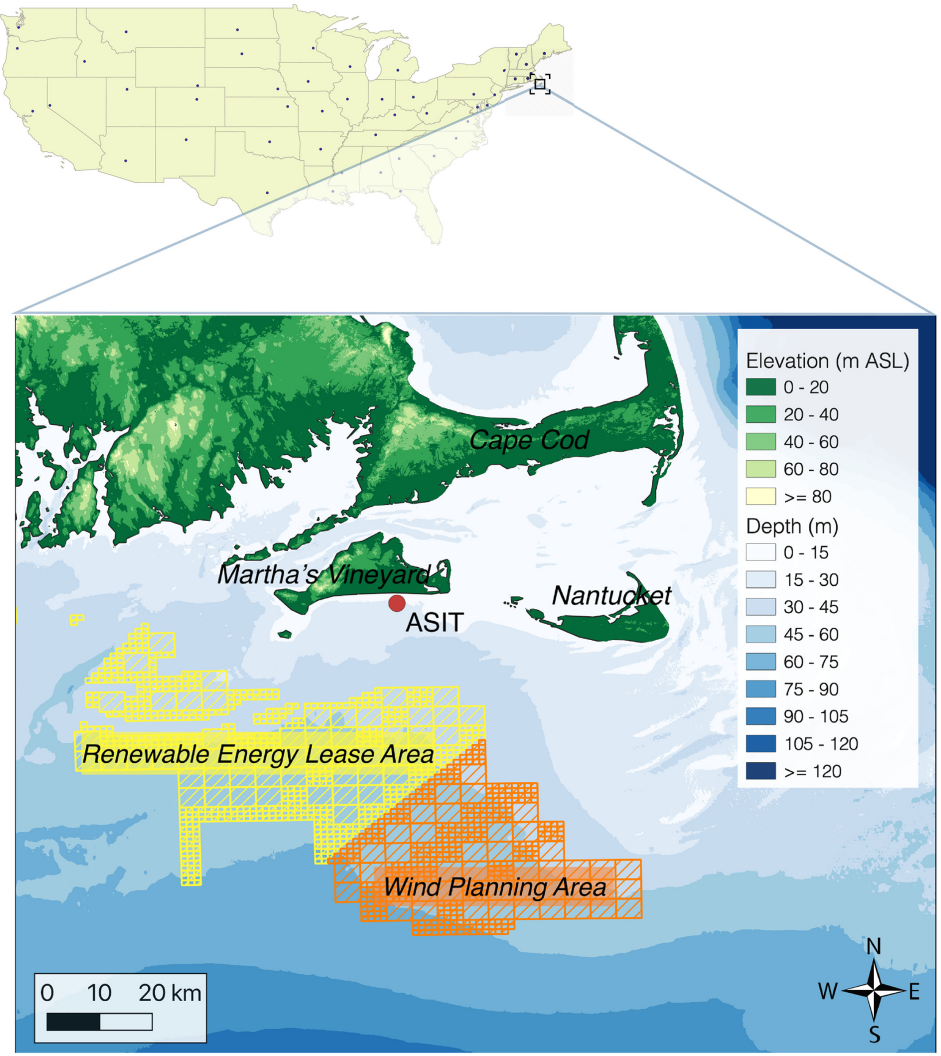


Application 3: Change Point Detection



Sequential change point detection is the online detection method on underlying change in the time series.

Ramp Detection on Renewable Energy Data



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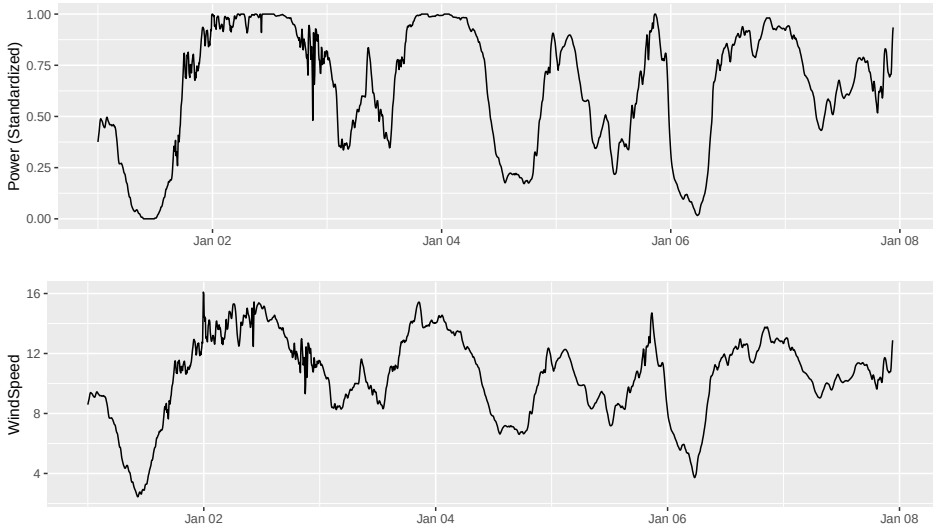

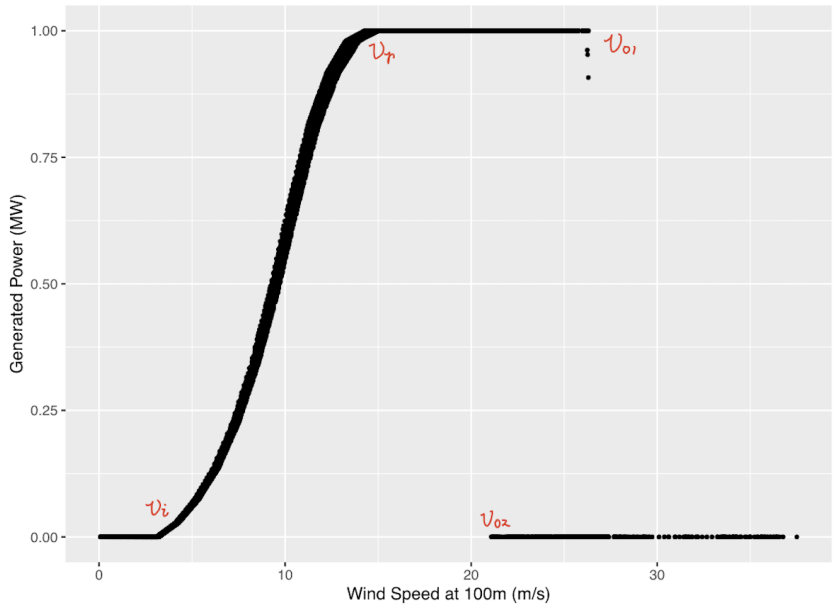

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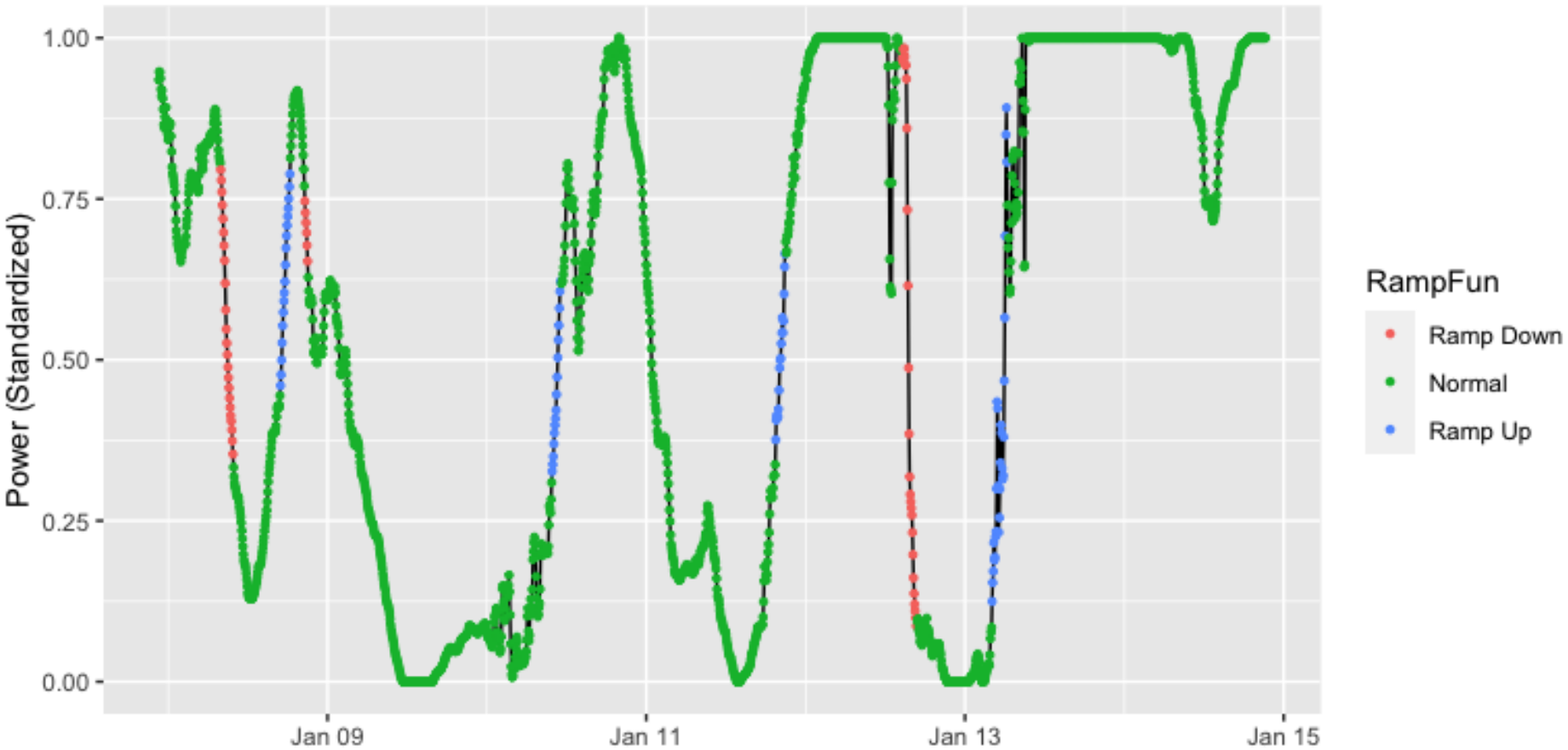

Comments

- Sequential sampling will be needed when there is no prior information on the data. e.g. variance is unknown.
- Sequential sampling will outperform fixed size sampling when sampling is costly.
- Sample size will affect the power of the analysis.
- In most cases, it is efficient to use G*Power to calculate the fixed sample size, or conduct power analysis.